

# Power in Forming Government Coalitions: An Algorithmic Approach

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## 1 Abstract

In this capstone we use an algorithmic approach to examine voting power in forming coalition governments. Specifically, we use the Shapley-Shubik power index on 61 election cycles in Germany, Japan, and the United Kingdom, and explore the relationship between vote share, seat share, and power. We also study the differences under this framework between the three countries and their three electoral systems: first-past-the-post, parallel voting, and mixed-member proportional. We find several differences between the countries, but find that in each, power correlates with seat share and voting share but is much more volatile, leading to disparate effects on political parties.

## 2 Background

### 2.1 Coalition games

In this thesis, we study voting games, in the sense of game theory, where the players are the voters, and each voter may have a different voting weight. One example is the electoral college, where we can view states as voters and their voting weights as the number of electoral votes they have. A significant body of research exists on understanding the power of voters in these games. Our goal is to use these techniques to analyze the formation of coalition governments.

In about 80 countries, the government is formed by a coalition of political parties that together make up a majority of the seats in parliament. The coalition may be made up of one party, if that party holds a majority of the seats, or multiple parties otherwise. In forming these coalitions, each party acts as a single voter, with voting weight equal to the number of seats they hold. The final outcome often includes arrangements where larger parties are forced to share power with smaller parties. The impact of these governments is far-reaching, as they determine everything from trade policy to the handling of the COVID-19 crisis.

Although there has been past research on voting power with regard to weighted majority games such as the electoral college [7] [10], as well as past research on examining the formation of coalition games with bargaining models [1] [2] [6], to our knowledge, there has been no effort to apply the Banzhaf and Shapley-Shubik algorithms to the weighted majority game of forming a coalition government.

## 2.2 Mathematical model

We consider the formation of a coalition government as a **weighted majority game**  $G$ . A weighted majority game  $G$  is a tuple  $(N, v, \mathbf{w})$  where  $N$  is a set of players,  $v : 2^N \rightarrow \{0, 1\}$  is a characteristic function which takes as input a coalition and outputs a binary variable indicating if the coalition is a winning coalition or not, and  $\mathbf{w}$  is a vector that includes an associated weight  $w_i$  for each player  $i$ . A coalition will pass a vote of confidence and form a government if more than half of the representatives in parliament are part of the coalition. Therefore, we define the **quota**  $q$  as equal to  $\lceil \frac{\sum_{i \in N} w_i}{2} \rceil$ , and we say that a coalition  $c$  is **winning** if  $\sum_{i \in c} w_i \geq q$ . Then,

$$v(N') = \begin{cases} 1 & \text{if } \sum_{i \in N'} w_i \geq q \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Although the notion of voting power applies more broadly, we describe the idea in the context of coalition government formation, the focus of this thesis.

There are several ways to calculate the power of a player in a weighted majority game. The two most widely-known are the Banzhaf Power Index [3] and the Shapley-Shubik Power Index [9]. As such, we intend to use these power indices.

Given a weighted majority game, a **power index**  $\mathbf{p}$  computes a vector  $\mathbb{R}^N$  that contains a weight for each player  $i \in N$ , which we interpret as its power. In particular,  $p_i(G)$  is the power of player  $i$  in the weighted majority game  $G$  under power index  $\mathbf{p}$ .

There are two main power indices for interpreting power in weighted majority games, the Banzhaf power index and the Shapley-Shubik power index. We choose the Shapley-Shubik as our measure of power because it emphasizes the power of the last voter to join the coalition, which is vitally important in real-life coalition forming. However, both are reasonable interpretations of power, so the Banzhaf power index is included in Appendix A. The raw results of the Banzhaf power index are also included alongside this report, as are the raw results of the Shapley-Shubik power index.

Here's the Shapley-Shubik index:

### Shapley-Shubik Power Index:

$$p_i^{ss}(N, v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|! (|N| - |S| - 1)!}{|N|!} (v(S \cup \{i\}) - v(S)) \quad (2)$$

The Shapley-Shubik power index, in words, calculates power as follows. It evaluates every permutation of voters in the game, and for some player  $i$  in the game, tallies how many times that player is the median voter. For example, in the sample game shown in section 2.3.1, if the permutation is Orange Party - Purple Party - Yellow Party, Purple party is the median voter. Then, the Shapley-Shubik power of some voter  $i$  is the number of times it is the median voter in a permutation divided by  $2^n$ , where  $n$  is the number of players. Note that  $2^n$  is also the total number of permutations of the  $n$  players, and each

permutation has exactly one median voter, so the sum of the Shapley-Shubik power of every player in the game is equal to 1.

### 2.2.1 Simple example

Consider a simple, imaginary example of a parliament that is in the process of forming a coalition government, for the purpose of understanding voting power. See Table 1. There are 1000 total representatives, so  $q = 500$ . Although the Purple Party holds half as many seats as each of the other two parties, both the Banzhaf and Shapley-Shubik power indices report that it has power equal to the others in forming a coalition government. This makes sense intuitively, because no single party holds a majority of the seats but any two parties together can form a winning coalition, so all three parties are on equal footing.

	<b>Orange Party</b>	<b>Yellow Party</b>	<b>Purple Party</b>
<b>Elected representatives</b>	400	400	200
<b>Banzhaf Power Index (normalized)</b>	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
<b>Shapley-Shubik Power Index</b>	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Table 1: A simple coalition government weighted majority game

## 2.3 UK, Japan, Germany

We focus on three countries, each with a long history of democracy, and each with their own style of election.

### 2.3.1 Three election systems

Each of the three countries has its own flavor of election for parliament.

The United Kingdom uses a first-past-the-post electoral system. This means that the country is divided into districts, and each district elects exactly one representative to parliament. This is not a proportionally representative system - if one party wins 51% of the vote in each district, then they win 100% of the seats in parliament, despite only winning 51% of the overall vote. The United States house of representatives is another example of a first-past-the-post system. Past research has consistently shown that first-past-the-post electoral systems trend towards 2 dominant parties and few mid-sized or small parties [8].

Japan uses a parallel voting electoral system. Like the United Kingdom's system, Japan is divided into districts, which each elect one representative. In Japan's system, however, voters fill out two ballots, one for their district and one for a general party. Then, a share of parliament is filled by the representatives for each district and a share is filled by the outcome of the party vote. This system is more representative than first-past-the-post, but not fully representative. For example, if a party wins 51% of the district votes in each district and 51% of the party level vote, then they will win 100% of the

district-elected representatives and 51% of the nationally-elected representatives. After they are elected, both types of representatives serve together in parliament and have the same duties and powers.

Germany uses a mixed-member proportional system. Like Japan, Germany is divided into districts, and voters fill out two ballots, one for their district and one national level. Unlike Japan, however, the nationally-elected seats are filled not in proportion to the national-level votes, but so that the entire legislation ends up proportional to the national-level votes. This turns out, then, to be a proportionally representative system. For example, if some party gets 51% of the district-level votes and 51% of the national-level votes, then it will get 100% of the district-elected representatives and enough national-level representatives so that it gets 51% of the overall parliament - in this case, very few national-level representatives. Like Japan, after they are elected, both types of representatives serve together in parliament and have the same duties and powers.

### 2.3.2 Three countries

We have varying amounts of data for each country.

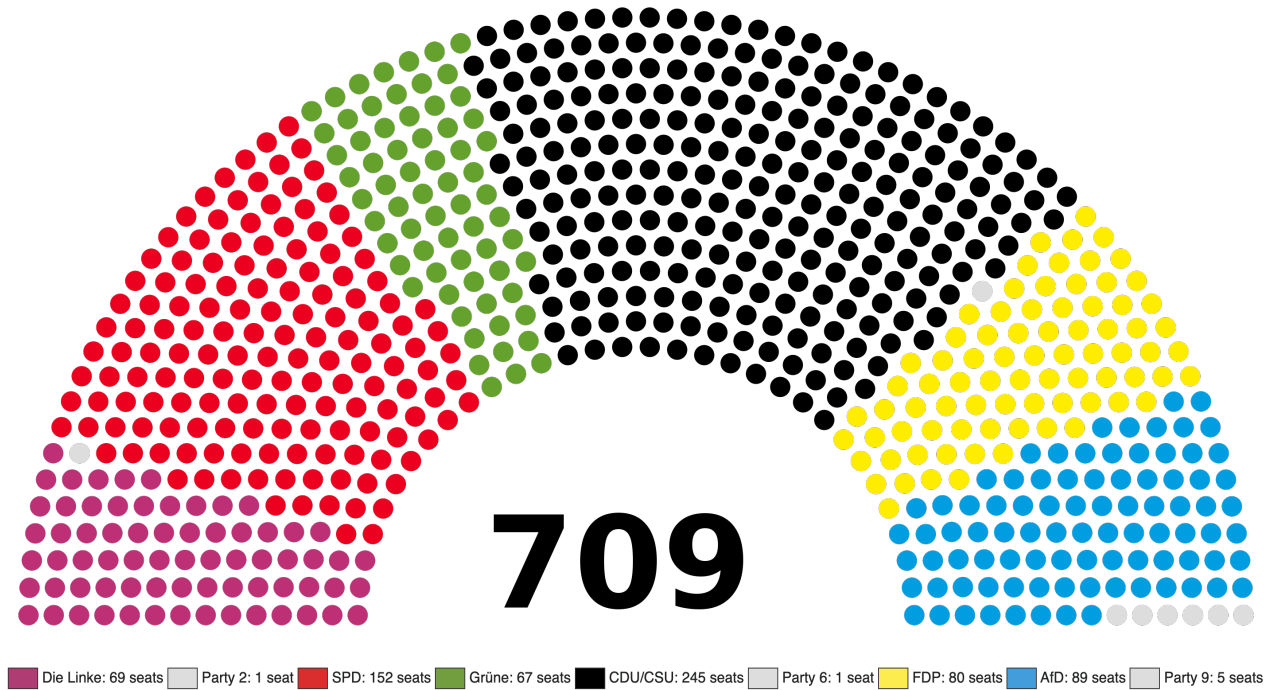
For the United Kingdom, we have data from the 28 election cycles from 1918 to 2019. There are 5 parties tracked in this dataset, but the major parties are the Conservative Party and the Labour Party. The Liberal Democrats also win a notable amount of seats in many elections. The United Kingdom is made up of about 650 districts, which elect Members of Parliament through their first-past-the-post system. Our data for the United Kingdom comes from the House of Commons Library website [5].

For Japan, we have data from 13 election cycles from 1969 to 2009. There are a large number of parties that form newly every election cycle, and a large number of mergers that happen between these parties - a quirk of Japan's system that is not the case in either the UK or Germany. There has been one dominant party, the Liberal Democratic Party, since 1969. The main opposition party until about 1993 was the Japan Socialist Party, and now is the Democratic Party of Japan. Japan's parliament, the National Diet, is made up of about 480 seats, of which 180 are proportionally elected and 300 are elected from districts. Our data for Japan comes from the Inter-Parliamentary Union's online archive of Japanese parliamentary election results [4].

For Germany, we have data from 19 election cycles from 1953 to 2021. There are a large number of parties that exist only briefly, often winning no representation at all. The party that has won the lion's share of the elections over the years is the Christian Democratic Union/Christian Social Union (CDU/CSU), which uses those two names in different parts of the country but is effectively one party. The main opposition party is the Social Democratic Party (SPD). The German parliament that determines the government is made up of around 600 seats, of which about 300 are from districts and about 300 are filled by parties to satisfy the proportionally representative nature of mixed member proportional voting. Our data comes from the Federal Returning Officer of Germany [11].

### 2.3.3 Real-world example

Consider the following real-world example. It is the current German parliament.



As can be seen in the graphic, there are 709 total representatives, so  $q = 355$ . The following table displays the results of the Shapley-Shubik index, run on this example.

	CDU	SPD	AfD	FDP	The Left	The Greens	Independent
<b>Representatives</b>	245	152	87	80	69	67	9
<b>Shapley-Shubik Power</b>	.4	.2	.1	.1	.1	.1	0

Table 2: Bundestag representatives, 2020.  $q = 355$

	CDU	SPD	AfD	FDP	The Left	The Greens	Independent
<b>Representatives</b>	311	193	0	0	64	63	0
<b>Shapley-Shubik Power</b>	.5	.17	0	0	.17	.17	0

Table 3: Bundestag representatives, 2014,  $q = 316$

These two examples reveal a few interesting things about the last two German governments.

First of all, in 2014, although SPD had more than 3 times as many votes as The Left or The Greens, they all had equal voting power - because the only way any of them could be part of the coalition government was to join with the CDU, or if every party but the CDU joined together. In this sense, they function as identical actors. Then, in 2020, although SPD lost 41 seats, it gained power, since the CDU lost seats and a number of other parties emerged. Conversely, The Left and The Greens both gained a small number of seats between 2014 and 2020 but lost power, since two competing parties emerged.

Also, looking at the independents in 2020, they have no power, even if they were to join together as a block. This likely plays a part in why there are so few independent representatives in the German system. If they join any other party, they gain power (and the party gains by adding members).

## 2.4 Objective

In studying these three countries, we intended to answer several questions:

- What is the relationship between vote share and power? Is that relationship affected by the election system - first-past-the-post, parallel voting, or mixed-member proportional?
- What is the relationship between seat share and power? Is that relationship affected by the election system?
- Are there parties that have gained a disproportionately large or small amount of power, compared with the share of votes they received?

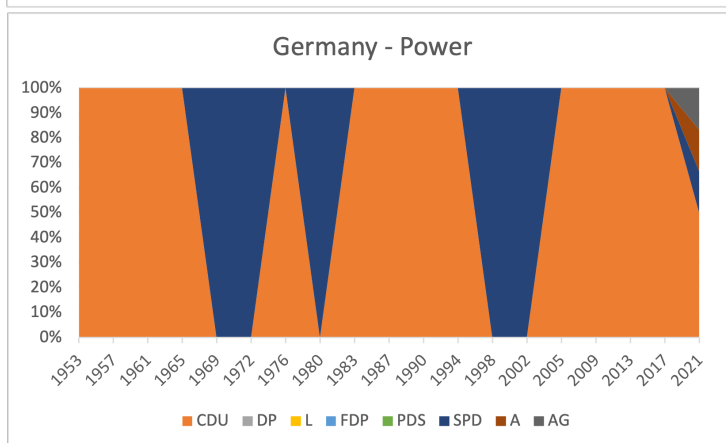
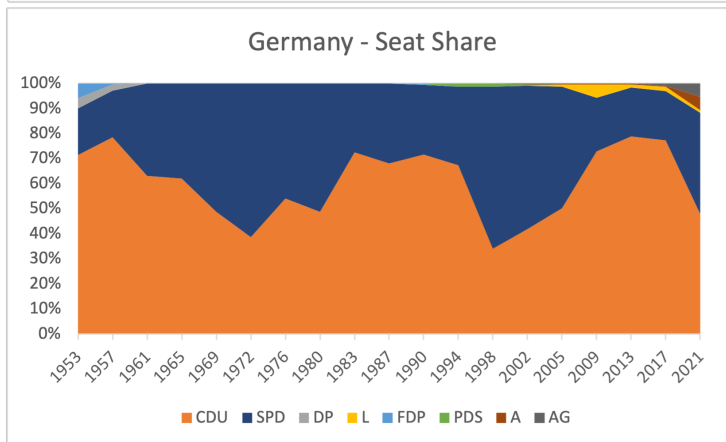
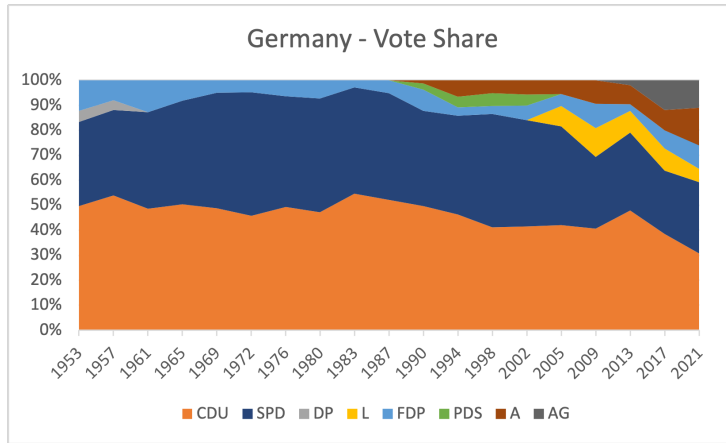
## 3 Methods

First, we collected data from online resources. For the UK, we took data from the House of Commons Library [5]. For Japan, we took data from the Inter-Parliamentary Union's online archive of Japanese parliamentary election results [4]. For Germany, we took data from the Federal Returning Officer of Germany [11]. All of the raw data is attached to this report.

After cleaning the data and organizing it into consistent formats, we wrote two scripts in Python, one that calculated the Banzhaf algorithm and one that calculated the Shapley-Shubik algorithm. We then ran both of these scripts on every election in the dataset. Those scripts are attached to this report, as are the raw outputs for both the Banzhaf and Shapley-Shubik algorithms. Finally, I reorganized the data in excel and built graphs to show various relationships.

# 4 Results and Discussion

## 4.1 Germany



Key:

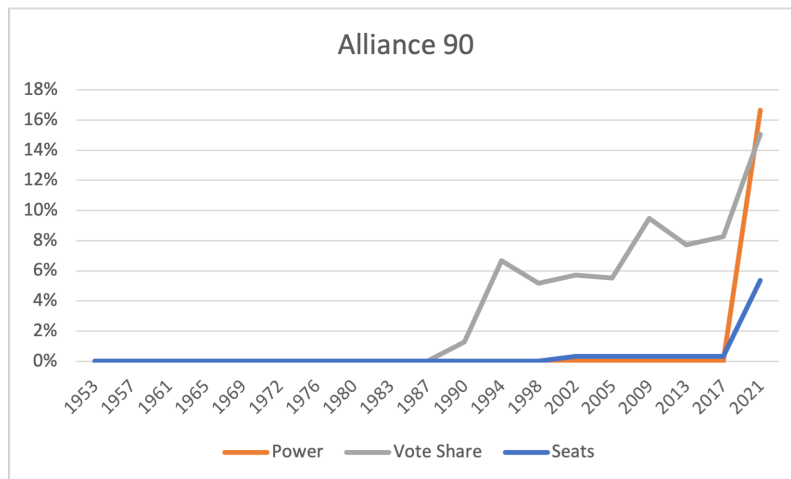
CDU	DP	L	FDP
Christian Democratic Union	German Party	The Left	Free Democratic Party
PDS	SPD	A	AG
Party of Democratic Socialism	Social Democratic Party	Alliance 90/The Greens	Alternative for Germany

## Discussion

As can be seen with all three countries, vote share does clearly correlate with seat share and power. However, small variations in vote share lead to large variations in seat share, which lead to even larger variations in power, often from 0 to 100 or 100 to 0.

Germany is unique in that every single election has led to one party controlling a majority of parliament, except for 2021. In that vein, utilizing the Shapley-Shubik power index does not gain us much - it does not take an algorithm to come to the conclusion that winning a majority of the seats in parliament leads to complete control of the government.

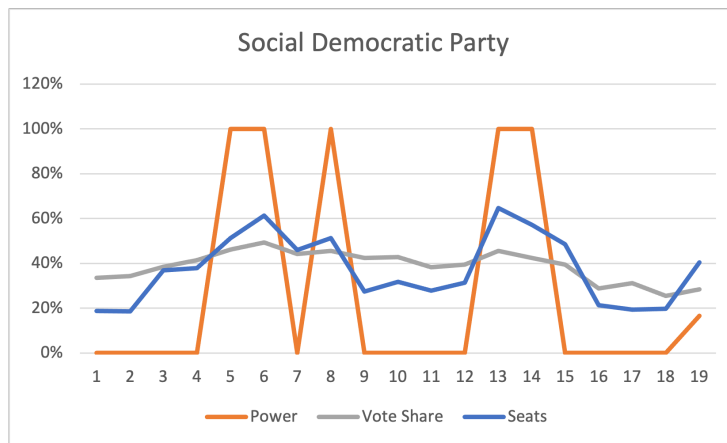
There aren't many small parties in Germany before 1990. However, since 1990, the number of those small parties and their influence has been growing. Although their gains in vote share have not translated to significant gains in seat share, in 2021 some of the small parties did gain power. Due to a decline in the seat share of the CDU/CSU to just below 50%, the small parties were positioned to possibly play a crucial part in forming a coalition government. As a result, they actually achieve outsized power in the 2021 election. One of these parties is Alliance 90. Here's a graph of its vote share, seat share, and power over the last 70 years.



As can be seen in the graph, Alliance 90 consistently got 5-10% of the overall vote share from 1990 to 2021. However, they didn't get nearly any seats until 2021. In 2021, their vote share jumped up to about 15%, leading to them getting about 5% of the seats and 17% of the power. It seems that they were sitting just below the threshold of getting a significant number of seats for much of their history, and they just passed it in 2021. However, as with many of the other small parties, they have had a relatively small amount of power given the number of votes they've received. Since 1990, Alliance 90 has accounted for 7.2% of the total votes cast, but only .8% of the seats and 1.9% of the power. Other parties have fared even worse: since 2005, The Left has accounted for 8.5% of the total votes cast but 2.0% of the seats and none of the power.

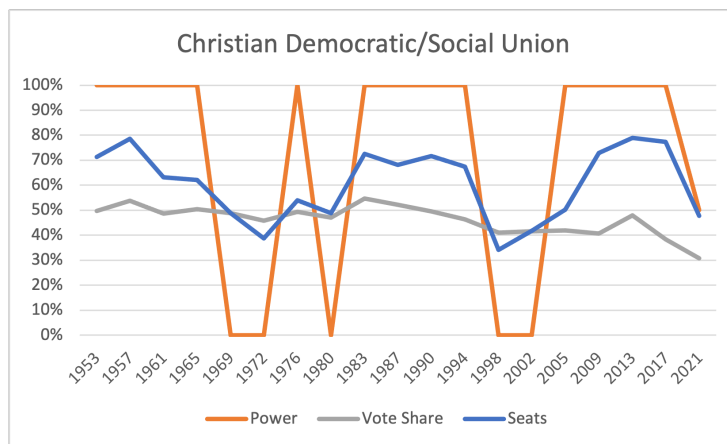
A larger party that has fared similarly is the Social Democratic Party.





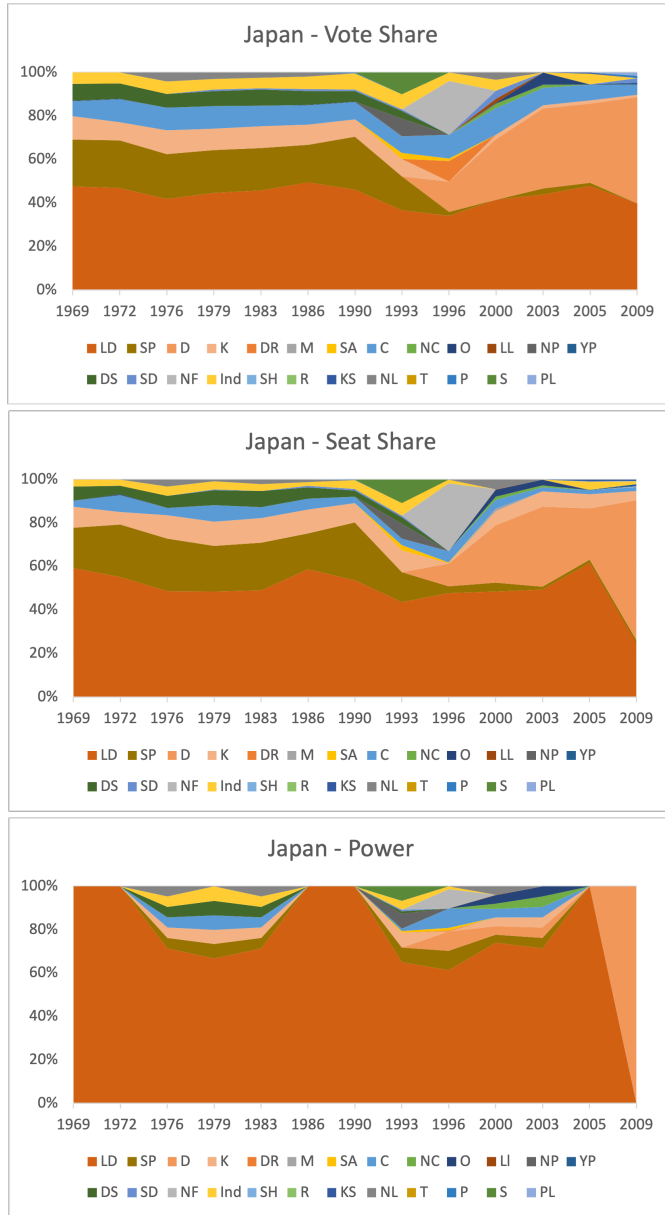
Although the SPD received 39% of the total votes cast since 1953 and took 38% of the seats, they only held 27% of the power. However, the power did not ever match the number of votes received or the number of seats - generally, they either reached just over 50% of the seats, leading to all of the power, or they reached just under 50% of the seats and the CDU/CSU reached just over 50% of the seats, leading to the SPD reaching no power. So, even though their overall power somewhat matches their vote share and seat share, in individual elections they rarely ever matched.

The party that has benefited the most from this system is the Christian Democratic Union/Christian Social Union.



As can be seen in the above graph, the CDU/CSU has often taken 100% of the power. All in all, the CDU/CSU received 46% of the votes, took 60% of the seats, and 71% of the power. Although they only had on average 7% more votes than the SPD, they ended up with 42% more power. This is because their graph has similar properties as the SPD graph - they often had just over 50% of the seats, leading to 100% of the power, or just under 50% of the seats, leading to none of the power (as the SPD had just over 50%). However, they came out at just over 50% more often than not, leading to them having the lion's share of the power over the last seventy years.

## 4.2 Japan



### Key:

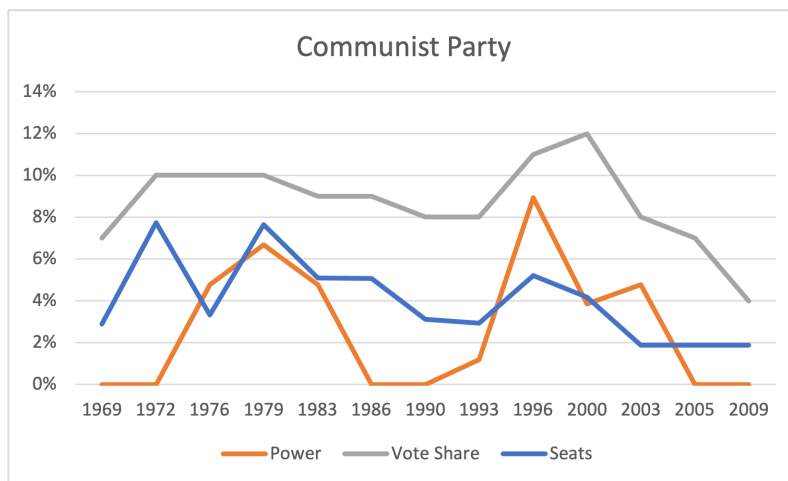
LD	SP	D	K	DR	M	SA
Liberal Democratic	Socialist Party	Democratic	Komeito	Democratic Reform	Mushozoku	Sakigake
C	NC	O	LL	NP	YP	DS
Communist	New Conservative	Other	Liberal League	New Party	Your Party	Democratic Socialist
SD	NF	Ind	SH	R	KS	NL
Socialist Democratic Federation	New Frontier	Independents	Shinseito	Renewal	Kokumin Shinto	New Liberal
T	P	S	PL			
Tomorrow	People's New	Shinto Daichi	People's Life			

### Discussion

As can be seen with all three countries, vote share does clearly correlate with seat share and power. However, smaller variations in vote share do lead to larger variations in seat share and power.

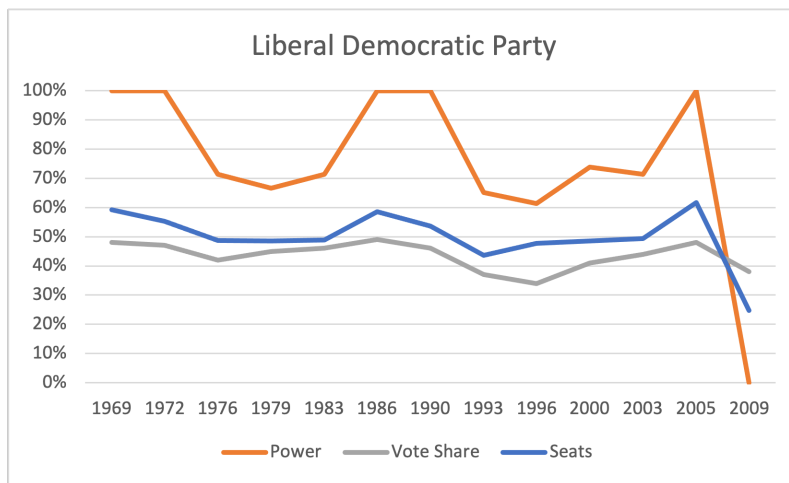
Japan is unique among the three countries we studied because it has been dominated by a single party, the Liberal Democrats, over the past 50 years. Here, the Liberal Democrats had a majority of the power in every single election cycle but the 2009, when they had a sharp decline and the Democratic Party of Japan took total power. Looking back, it took only a relatively small dropoff of vote share, to just below 40%, before they lost all power. However, there were situations where they had a similar amount of the vote, notably in 1993 and 1996, but still kept the majority of the power. It seems that having their opposition be fractured helped the Liberal Democrats keep power, but once their opposition was largely united as a single party, the Democratic Party of Japan, they lost power. This makes intuitive sense, too - in forming a coalition, if one party has close to half of the seats and there are many smaller parties that could bargain with them to push them over the threshold of 50%, then that large party still has much power. But if there is one opposition party that has over half of the seats, then the party that has just less than half of the power has no possible way to be a part of a winning coalition, and thus no power.

In this Japanese system (except for in 2009), it seems that there is a possibility of gaining power even as a small party, which explains why there are so many small parties, and why there are so many new parties. Here's one example, the Japan Communist party.



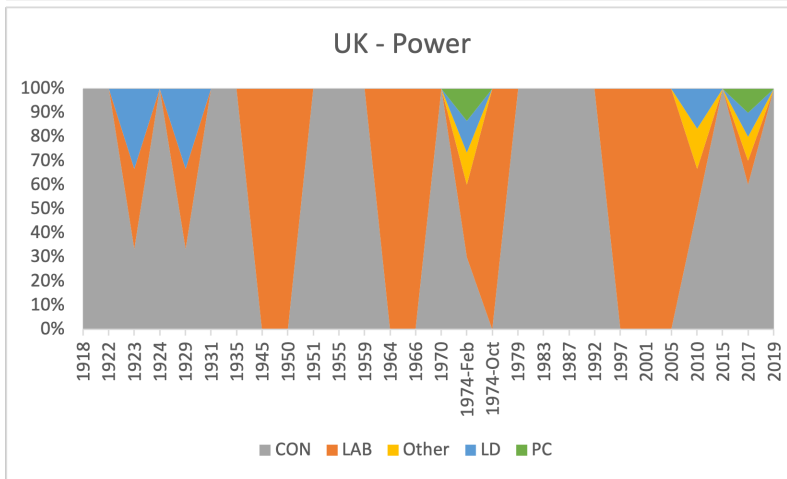
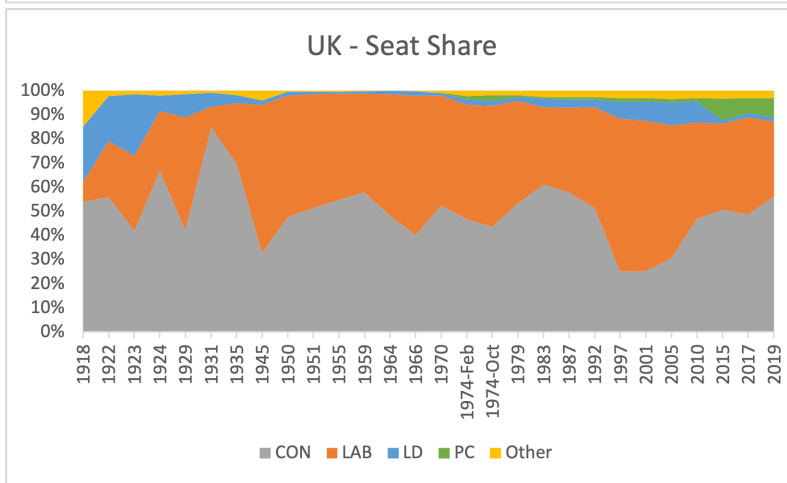
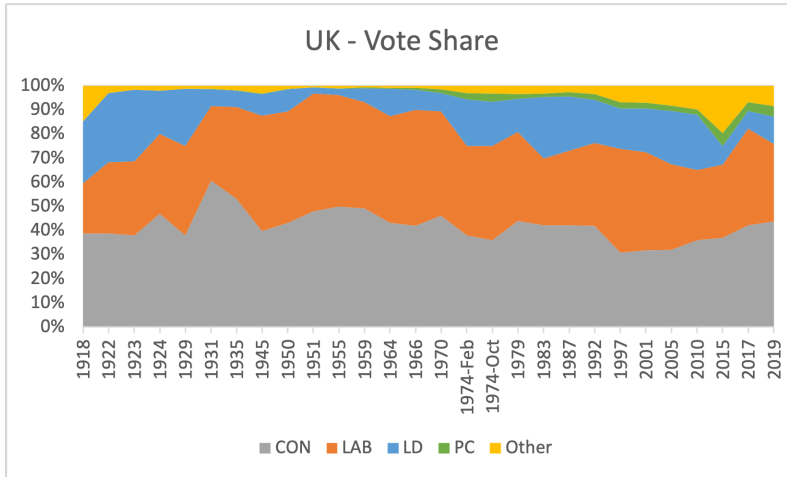
The Communist party remained a small party, but was still able to secure more than zero power in 7 of the 13 elections. Although they didn't get as much power as they got vote share, they still likely got more than a party in Germany or the UK with this share of votes would get.

Although this system seems to benefit small parties (at least relative to how much Germany and the UK's system benefits them), it most of all helps the major party.



The Liberal Democratic Party has received a hugely outsized share of power given their vote share, more than any other party in this analysis. Despite only receiving 37% of the votes across the 13 elections studied, they held on average 49% of the seats and 75% of the power. This 38 percentage point discrepancy between votes received and power received is enormous, and largely is accountable to the lack of a single strong opposition party, as discussed above. It is possible, if the Democratic party continues to exist as a strong opposition party, that this trend will change, and Japan will start to look more like Germany or the UK.

### 4.3 United Kingdom



Key:

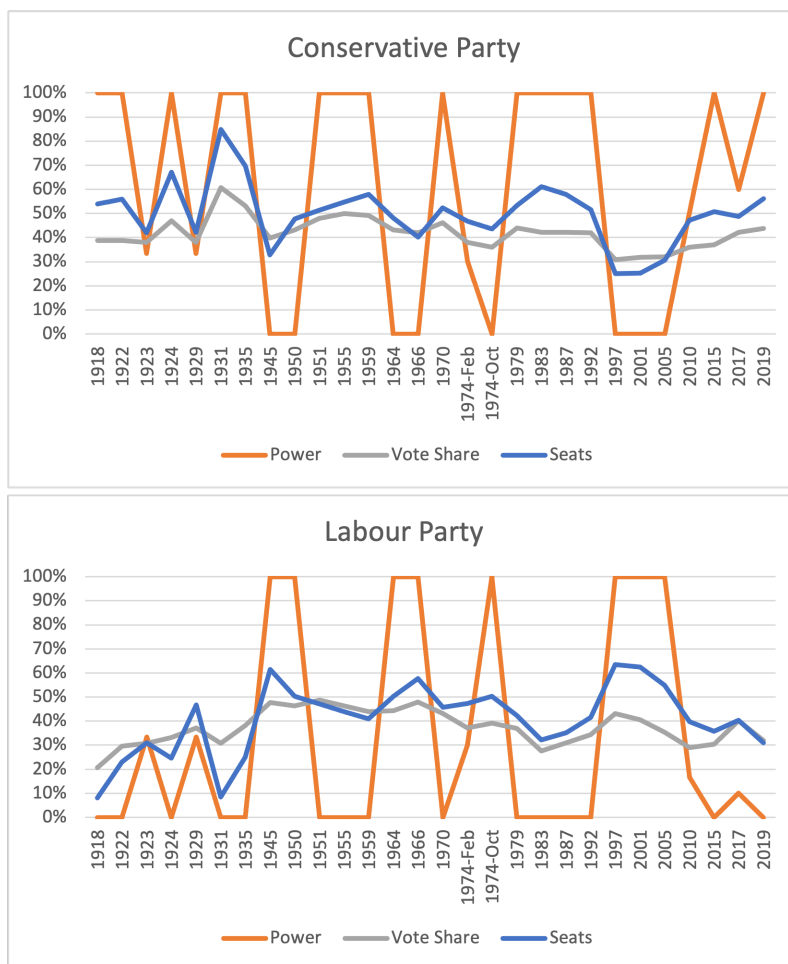
CON      LAB      LD      PC  
 Conservative   Labour   Liberal Democrats   Plaid Cymru

### Discussion

As can be seen with all three countries, vote share does clearly correlate with seat share and power. However, smaller variations in vote share do lead to larger variations in seat share and power.

The data source categorized all parties past the top four as "other," and this distinction makes sense, as they rarely get much vote share at all. Overall, the UK's graphs look more similar to Germany than to Japan, but small parties have even less of the vote share than in either other country. Just as was the case in Germany, two parties - in this case the Conservative Party and the Labour Party - hover with their seat share around 50%, and any time one of them gets over 50%, they get 100% of the power. However, there were a couple of elections where neither party had 50% of the seats, leading to scenarios where the Liberal Democrats had outsized power. Notably, this happened in 1922, 1929, and 2010.

Also similar to Germany, the party that has had a slight edge has had outsized power through the last 100 years.



Although the Conservative Party had 41% of the votes over the past 28 election cycles, which is only slightly higher than Labour's 37%, this had led to outsized representation. Over that same time frame, the Conservative party held 50% of the seats, compared with Labour's 41% of the seats, and held 61% of the power, whereas Labour only held 33% of the power. Thus, this 4 average percentage point

gap in votes between the two parties led to a gap of 28 point gap in average power. Similar to Germany, the smaller parties were vastly underrepresented. Of parties that were not the Conservative or Labour parties, they took 22% of the vote, 9% of the seats, and only 6% of the power. This backs up the existing literature that first-past-the-post systems trend towards two party systems and provides a reason why: smaller parties are underrepresented in the situation of forming a coalition government.

## 5 Conclusion

There are a number of inferences that can be drawn from the data above, but a few stand out as the most surprising and most impactful takeaways.

Firstly, in all three systems vote share, seat share, and power are correlated but have increasing volatility, so small changes in seat share lead to large changes in power. The most notable volatile situation is when a party has a seat share around fifty percent - with just over 50% they secure 100% of the power, a huge win and an over-representation of the votes they received. However, if they slip just below fifty percent, they risk significantly reduced power, especially the risk of going to zero if the opposition party secures the fifty percent threshold. This is why parties that often are just above the fifty percent threshold are generally overrepresented - Germany's Christian Democratic Union/Christian Social Union, Japan's Liberal Democrats, and the UK's Conservatives - while the parties that are just below the fifty percent threshold are underrepresented - Germany's Social Democratic Party and the UK's Labour party. Likewise, small parties, such as Germany's Alliance 90 or Japan's Communist Party, are vastly underrepresented.

Another theme that naturally arises from this data is the importance of the allocation of seats between your opponents. Japan's Liberal Democratic Party got 44% of the seats in 1993 and 25% of the seats in 2009, but ended up with 65% of the power in 1996 and 0% in 2009. This was largely due to their opponents in these elections. In 1993, there were a number of small parties, each with a small amount of power, but in 2009, the Democratic Party got 64% of the seats (despite only beating the Liberal Democrats by 8 points in vote share), taking all the power. In that vein, it is generally advantageous for your opposition to be split between a greater number of parties.

Finally, the three electoral systems seem to have disparate effects on incentives to form small parties. Germany seems to dissuade small parties the most (parties other than the CDU/CSU and SPD only comprise 15% of ballots cast), and this lines up with the power outcomes of their system. The only time in the last 70 years that one of the smaller parties has had any power at all was in 2021. The UK also seems to dissuade small parties (22% of the vote has gone to parties other than Conservatives and Labour) but provides more opportunities for small parties to seize power, as there are 5 elections of the past 28 where small parties have had some power. Japan, by far, has the most small parties (63% of all votes cast have gone to parties other than the Liberal Democrats), and this lines up with their power outcomes, too. In the majority of their elections a small party has had some power, and of those elections, in each of them about 5 small parties have had at least some power. See Appendices B and

C for more examples. Thus, it makes sense that Japan has so many small parties. 25 parties have ever won a seat in Japanese parliament, while only 8 have ever won a seat in German parliament.

It's important to note that this is only one measure of political power. Forming the coalition government is just one duty of many that a parliament undertakes, and this is one algorithmic analysis of that situation. There is a lot more that goes on with parliaments besides forming the government, and as such a lot more work that must be done before we can say we have a complete understanding of who controls power in parliament.

But this analysis is certainly a step in the right direction. These varying governmental frameworks affect what parties develop over time, how much power they hold, and, ultimately, what choices governments make. We all want a government that acts in good faith to represent the will of the voters, and understanding when systems do and don't represent the votes of the voters in forming a coalition government helps us understand why governmental policy outcomes are what they are, as well as how these systems can improve.

## 6 Future work

The data shown above is a starting point for further research. There are numerous directions for future research that could be fruitful. A few of potential future questions are below:

- This analysis only includes Germany, Japan, and the UK, which is a small sample set, especially since it is only one country of each electoral system. Examining a larger set of countries would no doubt give us greater insight.
- There are other methods of measuring power in forming coalition governments, such as the method of representing the situation as a bargaining game [6]. Comparing our results to these results would be interesting.
- Similarly, in other research the final cabinet assignments of coalition governments have been used as a ground truth for power [1]. Adding that layer to our analysis would be insightful.
- We could modify our algorithms to address the "formateur," the official who proposes the coalition. Other analyses have found that this formateur, who is usually a member of parliament chosen randomly or chosen by the head of state, holds extra power [2].



## 7 Appendices

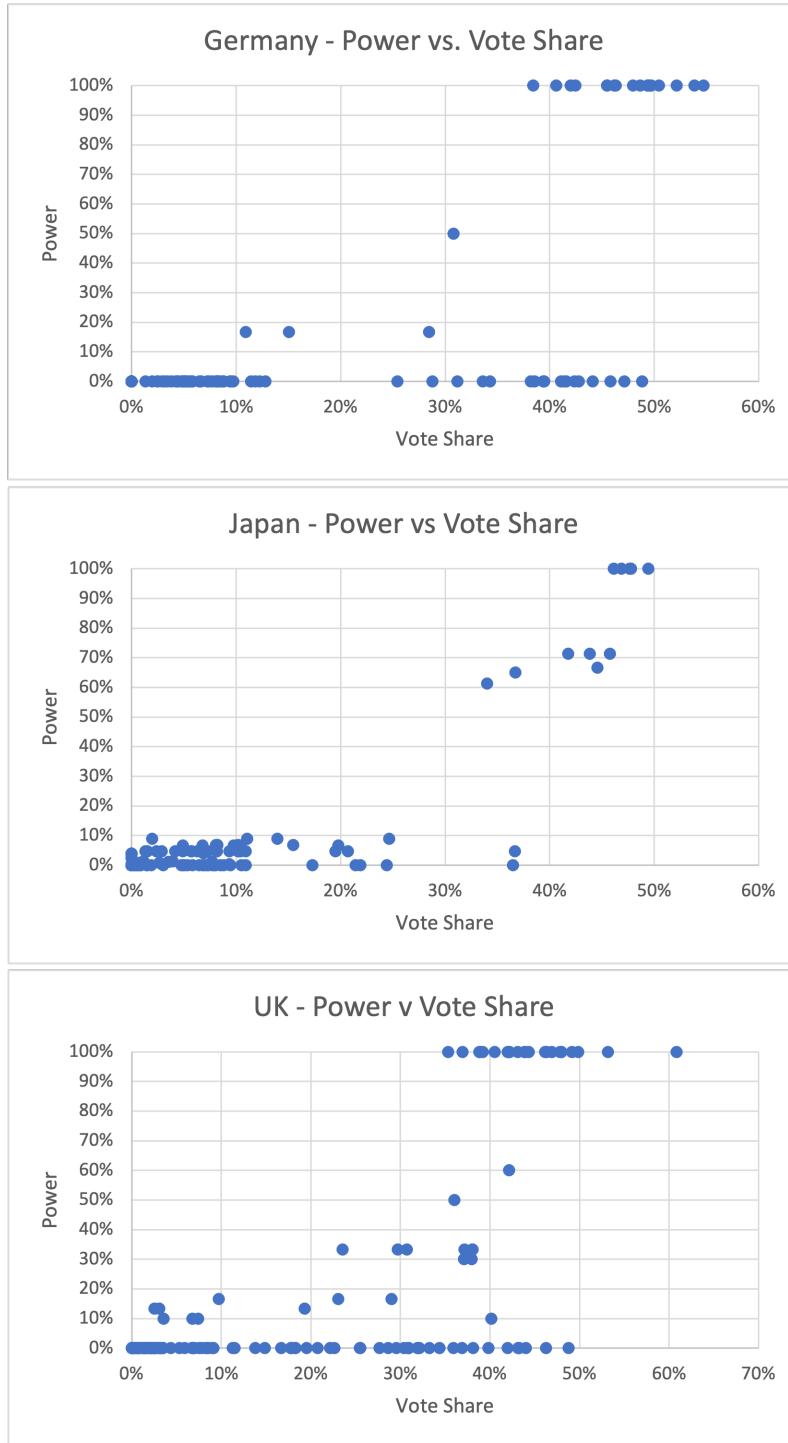
### Appendix A - The Banzhaf Power Index

**Banzhaf Power Index (normalized) [3]:**

$$p_i^{\text{bh}}(N, v) = \frac{\sum_{S \subseteq N \setminus \{i\}} (v(S \cup \{i\}) - v(S))}{\sum_{j \in N} \sum_{S \subseteq N \setminus \{j\}} (v(S \cup \{j\}) - v(S))}. \quad (3)$$

As discussed in section 2.2, the Banzhaf Power Index is a reasonable way to understand power in the situation of forming coalition governments. Although we choose to use the Shapley-Shubik algorithm, and those results are the ones displayed in all graphs and results above, we also have the results from the Banzhaf Power Index. That raw output is attached to this report.

## Appendix B - Vote Share vs Power



### Discussion

Although auxiliary to the main thrust of the report, graphs of power versus vote share can be helpful in further analysis.

First of all, it seems that in the UK and Japan, it is more likely that parties with a small share of

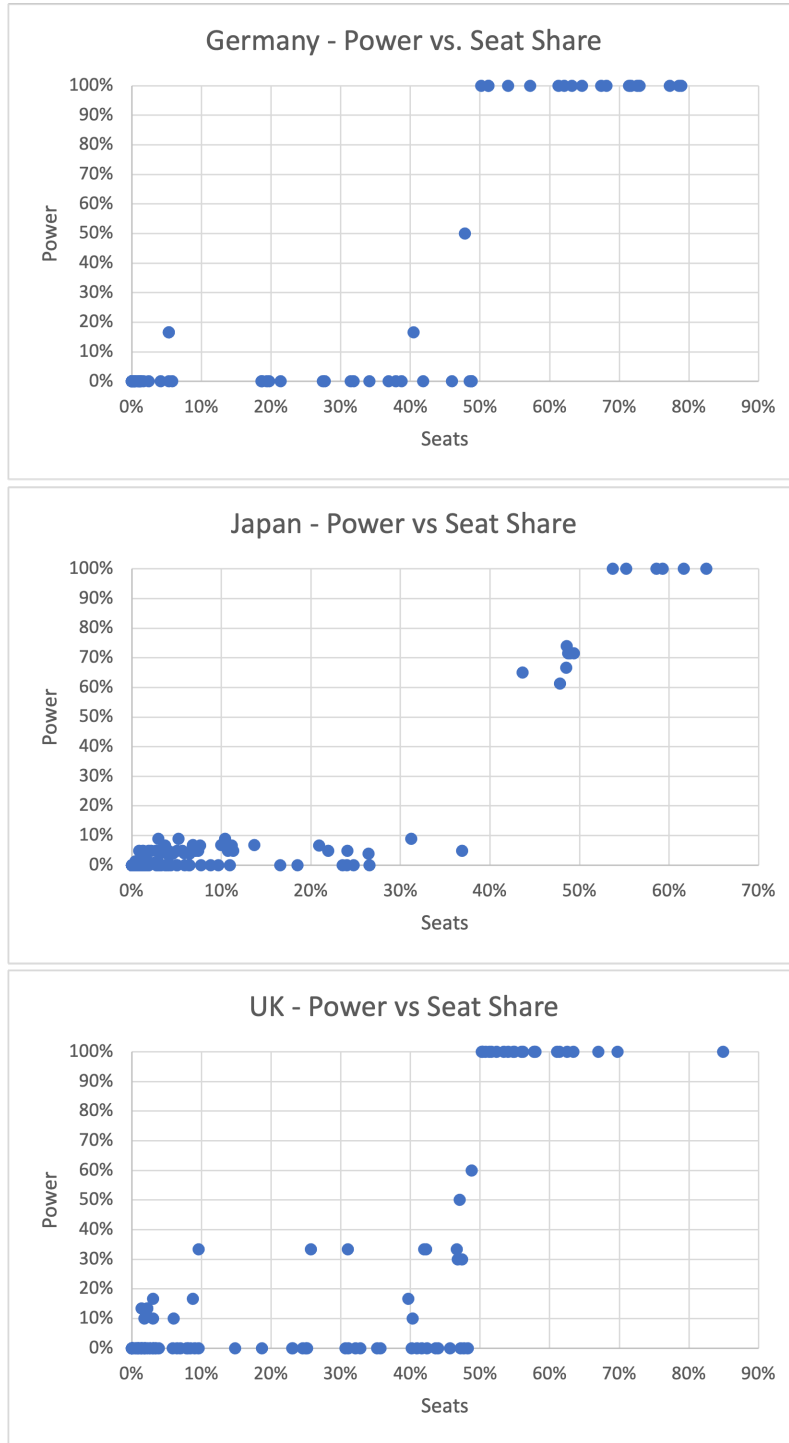
the vote will get some power at all. This is especially true in Japan, where there are many data points between 0 and 10 percent of the vote share and between 0 and 10 percent of the power - although it is still true that these parties are underpowered, relative to their vote share. This once again points to why Japan has so many small upstart parties.

Similarly, in Japan it seems nearly impossible to get the all of the power unless you get very close to 50% of the vote share. That is not as much the case in Germany and the UK, where there are examples of parties getting all of the power with less than 40% of the vote share.

In Germany and the UK, it seems more possible to get a higher percentage of the vote and still get no power at all.

It is important to note that we have limited examples and a small dataset, and running this same analysis with more elections would provide clearer results.

## Appendix C - Seat Share vs Power



### Discussion

Although auxiliary to the main thrust of the report, graphs of power versus seat share can be helpful in further analysis.

First of all, there is a clear cutoff at 50% for getting total power. This makes intuitive sense. If you

have more than half of the seats in parliament, you have enough to govern without any coalition, but if you have fewer than half, you must form a coalition with another party.

Once again, it seems that in Japan (and somewhat in the United Kingdom) it is more likely that a party with a small percentage of seats, generally between 0 and 10, will still get some power. This again points at why Japan has so many small parties.

In Germany, it is very rare to get anything other than 0% or 100% of the power, as there is almost always a party with an outright majority.

In Japan, either a party gets less than 10% of the power (generally if they get less than 40% of the seats), or a majority of the power (generally if they get more than 40% of the seats), with nothing in between. This is due to their one-party dominant system - that party (or, in 2009, the Democratic party) always gets a majority of the power, and every other party gets a small share.

## References

- [1] Y Adachi T; Watanabe. “Ministerial weights and government formation: Estimation using a bargaining model.” In: *The Journal of Law, Economics, Organization* 24.1 (2008), pp. 95–119. DOI: <http://dx.doi.org/10.1093/jleo/ewm040>.
- [2] S; et al. Ansolabehere. “Voting weights and formateur advantages in the formation of coalition governments.” In: *American Journal of Political Science* 49.3 (2005), pp. 550–563. DOI: <https://doi.org/10.2307/3647731>.
- [3] John F. Banzhaf. “Weighted Voting Doesn’t Work: A Mathematical Analysis”. In: *Rutgers Law Review* 19.2 (1965), pp. 317–343.
- [4] M Chungong. “Historical Archive of Parliamentary Election Results”. In: *House of Commons Library* (2014). DOI: [http://archive.ipu.org/parline-e/reports/2161\\_arc.htm](http://archive.ipu.org/parline-e/reports/2161_arc.htm).
- [5] S Cracknell R; Pilling. “UK Election Statistics: 1918-2021: A century of elections”. In: *House of Commons Library* (2021). DOI: <https://commonslibrary.parliament.uk/research-briefings/cbp-7529/>.
- [6] H Diermeier D; Eraslan. “A structural model of government formation”. In: *Econometrica* 71.1 (2003), pp. 27–70. DOI: <http://dx.doi.org/10.2139/ssrn.305881>.
- [7] O de Mouzon. “One Man, One Vote Part 1: Electoral Justice in the U.S. Electoral College: Banzhaf and Shapley/Shubik Versus May”. In: *Studies in Choice and Welfare* (2020). DOI: [https://doi.org/10.1007/978-3-030-48598-6\\_9](https://doi.org/10.1007/978-3-030-48598-6_9).
- [8] H Riker. “The Two-party System and Duverger’s Law: An Essay on the History of Political Science”. In: *American Political Science Review* 76.4 (1982), pp. 753–766. DOI: <https://doi.org/10.1017/S0003055400189580>.
- [9] M Shapley L.S.; Shubik. “A Method for Evaluating the Distribution of Power in a Committee System”. In: *American Political Science Review* 48.3 (1954), pp. 787–792. DOI: [10.2307/1951053](https://doi.org/10.2307/1951053).
- [10] R Sickles. “The Power Index and the Electoral College: A Challenge to Banzhaf’s Analysis”. In: *Villanova Law Review* 14.1 (1969), pp. 86–92. DOI: <https://doi.org/10.1017/S0003055400189580>.
- [11] G Thiel. “Elections to the German Bundestag: Election Data”. In: *Election Resources on the Internet* (2021). DOI: <http://electionresources.org/de/data/>.